Gap-filling based on iterative EOF analysis of the temporal covariance: application to InSAR displacement time series

Alexandre Hippert-Ferrer[1], Yajing Yan[1], Philippe Bolon[1]

[1] Laboratoire d'Informatique, Systèmes, Traitement de l'Information et la Connaissance (LISTIC), Annecy, France

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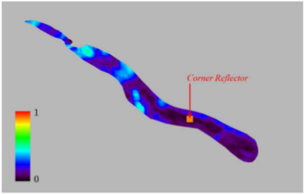
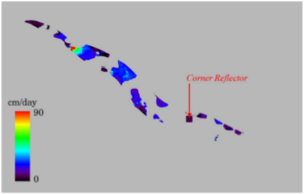
Content

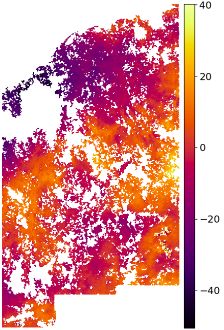
Introduction

- Missing data is a frequent issue in SAR-derived products in both space and time.
- It can prevent the full understanding of the physical phenomena under observation.
- Causes : rapid surface changes, missing image, technical limitations.





Argentiere glacier, offset tracking of TerraSAR-X in Summer 2010 [2]



Interferogram over land area, Mexico (Isterre)

Motivation of the study

Handling missing data in InSAR displacement time series

- Classical approach : spatial interpolation
- Not exploited (yet) : temporal information

→ Manage spatio-temporal missing data in time series ←

Proposed : a statistical gap-filling method addressing

1. Randomness and possible space time correlation of
   - Noise
   - Missing data
2. Mixed frequencies displacement patterns (complex signals)

Expectation Maximization-Empirical Orthogonal Functions

Key components of the proposed method :

- Signal learned as empirical orthogonal functions (EOFs).
- Low rank structure of the sample temporal covariance matrix.
- Reconstruction with appropriate initialization of missing values [1].
- Expectation-Maximization (EM)-type algorithm for resolution.

1. [1] Beckers and Rixen, "*EOF calculations and data filling from incomplete oceanographics datasets,*" J. Atmos. Oceanic Technol., vol.20(12), pp.1836-1856, 2003

EM-EOF : data representation and initialization

- Let $X(\mathbf{s}, t)$ be a spatio-temporal field containing the values of X at position $\mathbf{s}$ and time t :

$$X = (\mathbf{x}_1, \mathbf{x}_2, \dots \mathbf{x}_n) = \begin{pmatrix} x_{11} & x_{12} & x_{13} & \cdots & x_{1n} \\ x_{21} & x_{22} & x_{23} & \cdots & x_{2n} \\ x_{31} & x_{32} & x_{33} & \cdots & x_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_{p1} & x_{p2} & x_{p3} & \cdots & x_{pn} \end{pmatrix}$$

$(x_{ij})_{1 \leq i \leq p, 1 \leq j \leq n}$ is the value at position $\mathbf{s}_i$ and time t_j and may be missing.

- Missing values are then initialized by an appropriate value (*first guess*).

EM-EOF : covariance estimation and decomposition

- The sample temporal covariance is first estimated :

$$\boxed{\hat{C} = \frac{1}{p-1}(X - \mathbf{1_n}\bar{X})^T(X - \mathbf{1_n}\bar{X})}$$

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- EOFs $(\mathbf{u}_i)_{0 \leq i \leq n}$ are the solution of the eigenvalue problem :

$$\boxed{\hat{C}U = U\Lambda}$$

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- EOFs can be used to express $\hat{C}$ in terms of EOF modes :

$$\boxed{\hat{C} = \lambda_1 \mathbf{u}_1 \mathbf{u}_1^T + \lambda_2 \mathbf{u}_2 \mathbf{u}_2^T + \cdots + \lambda_n \mathbf{u}_n \mathbf{u}_n^T}$$

EM-EOF : reconstruction of the field

- X' is reconstructed with M number of EOFs :

$$\boxed{X' = \sum_{i=1}^{n} a_i \mathbf{u}_i^t \rightarrow \hat{X}' = \sum_{i=1}^{M \ll n} a_i \mathbf{u}_i^t}$$

with $a_i = X'\mathbf{u}_i$ are the Principal Components (PCs) of the anomaly field (X').

- The first EOF modes capture the main temporal dynamical behavior of the signal whereas other modes represent various perturbations [2].
- **Goal** : find the optimal M

2. [3] R. Prébet, Y. Yan, M. Jauvin and E. Trouvé, "*A data-adaptative EOF based method for displacement signal retrieval from InSAR displacement measurement time series for decorrelating targets*", IEEE Trans. Geosci. Remote Sens., vol. 57(8), pp. 5829-5852, 2019

Cross-validation

- **Cross-RMSE** : cross-validation root-mean-square error :

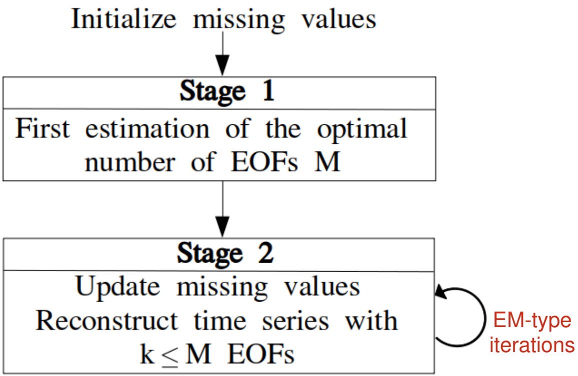
$$\boxed{E(k) = \left[\frac{1}{N} \sum_{k=1}^{N} |\hat{\mathbf{x}}_k - \mathbf{x}|^2 \right]^{1/2}}$$

k : number of EOF modes used in the reconstruction

- **Key parameter** : the optimal number of EOF modes M, estimated by :

$$\boxed{\underset{M \in [1,n]}{\arg\min}\, E(k)}$$

A 2-stage method



Numerical simulations : setup

- Displacement fields with different complexity are computed :

	$g(r,t)$	Order
g_1	$(1-0.5r)t$	1
g_2	$g_1+\sin(2\pi f_1 t)\cos(2\pi f_1 r)$	2
g_3	$g_2+0.5\cos(2\pi f_2 t)\cos(2\pi f_3 r)$	3
g_4	$g_3+0.1\sin(2\pi f_4 t)\cos(2\pi f_5 r)$	4

TABLE – $f_1=0.25$, $f_2=0.75$, $f_3=2.5$, $f_4=1.25$, $f_5=5$.

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- Type of noise : random $\sim \mathcal{N}(0,1)$, spatially and spatio-temporally correlated
- Type of gaps : random, correlated

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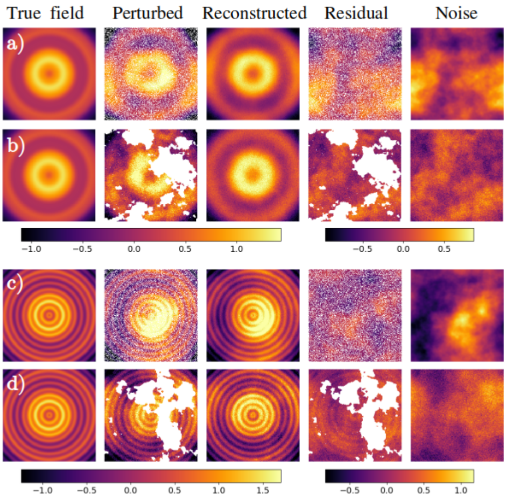
- Type of noise : random $\sim \mathcal{N}(0,1)$, spatially and spatio-temporally correlated
- Type of gaps : random, correlated
- SNR = [0.5,4.5]
- Gaps [0,80]%
- Initialization value : spatial mean, spatial mean + random noise, spatial mean + correlated noise

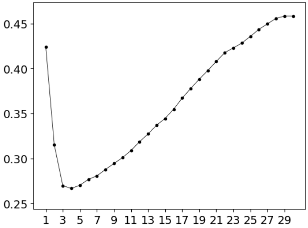
Numerical simulations : 3rd and 4th order fields



SNR and % of gaps are fixed :

- SNR = 1.5
- 30% of gaps

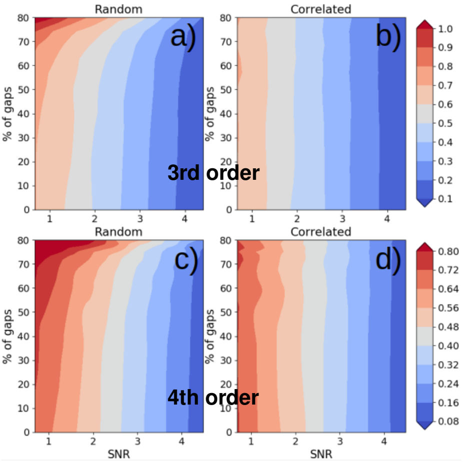
Number of EOF modes vs. cross-RMSE :



→ Minimum of the error found at the signal order

Sensitivity to SNR and % of gaps

Cross-RMSE in function of % of gaps and SNR :



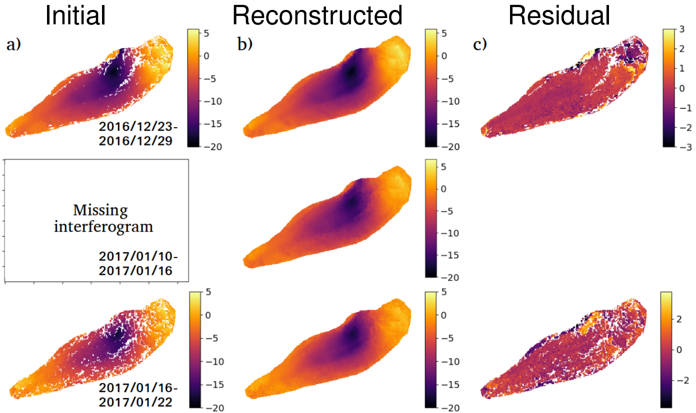
- The method is more sensitive to SNR than to the % of gaps
- Random gaps affect more the reconstruction than correlated (seasonal) gaps
- Initialization value affects the time of convergence but not the estimation of M

Data and area of study

Glacier	Period	Platform	Data type	Size	[min, max]% missing	Missing images
Gorner	11/2016-03/2017	Sentinel-1/A	Interferometry	16	[11.8, 27.8]%	4
Miage	12/2016-03/2017	Sentinel-1/A	Interferometry	13	[11.4, 23.1]%	3
Argentière	10/2016-12/2017	Sentinel-1/A/B	Offset tracking	65	[2, 50]%	0

TABLE – Time series description.

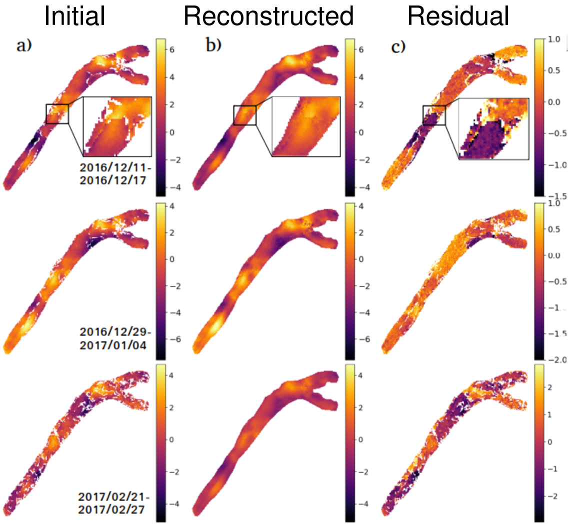
Case 1 : Gorner Glacier



A. Hippert-Ferrer, Y. Yan and P. Bolon, Em-EOF : gap-filling in incomplete SAR displacement time series, 2019, submitted.

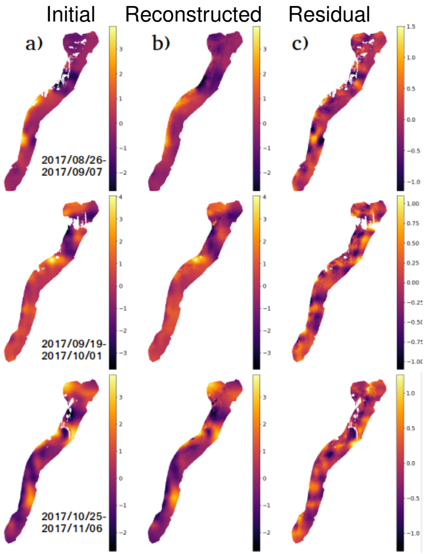
- Number of EOF modes : 3
- Consistent pattern in missing data areas
- Missing interferogram is reconstructed by adding the temporal mean to the anomaly.

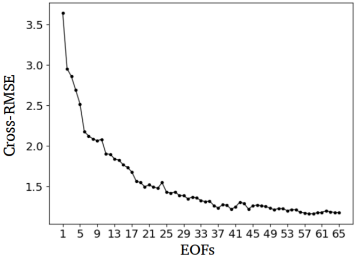
Case 2 : Miage Glacier



- Number of EOF modes : 2
- Discontinuities in the residuals due to phase jumps in the original interferogram.
- Detection and correction of inconsistencies.

Case 3 : Argentière Glacier





- Very low SNR and strong correlated gaps in space and time
- Strong mixing between displacement signal and noise
- Global agreement between reconstructed and initial fields

Conclusion

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- La méthode **EM-EOF** peut prendre en charge des cas complexes
  - Interférogrammes manquants
  - Discontinuités dues aux sauts de phase (perte de coherence)
- Apte à augmenter la taille effective d'un jeu de données
- Limitations de la méthode
  - Plus More sensitive to SNR than to % of gaps.
  - Argentière case : some breakdown points → potential for improvement

Perspectives

- Applications : slow slip event, glacier velocities from optical data...
- Estimation of a covariance matrix with missing data
- Time series of complex interferograms before unwrapping

Questions

Thank you for your attention.

[1] J. M. Beckers and M. Rixen. EOF calculations and data filling from incomplete oceanographics datasets. **J. Atmos. Oceanic Technol.**, 20(12) :1836–1856, 2003.

[2] R. Fallourd, O. Harant, E. Trouvé, and P. Bolon. Monitoring temperate glacier displacement by multi-temporal TerraSAR-X images and continuous GPS measurements. **IEEE J. Sel. Top. Appl. Earth Obs. Remote Sens.**, 4(2) : 372–386, 2011.

[3] R. Prébet, Y. Yan, M. Jauvin, and E. Trouvé. A data-adaptative eof based method for displacement signal retrieval from insar displacement measurement time series for decorrelating targets. **IEEE Trans. Geosci. Remote Sens.**, 57(8) :5829–5852, 2019.

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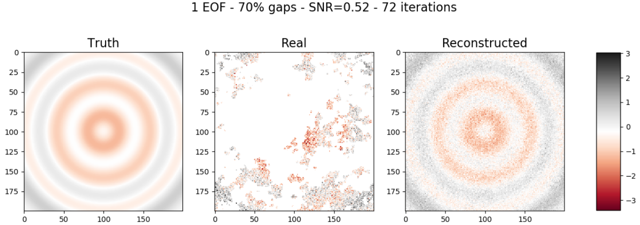
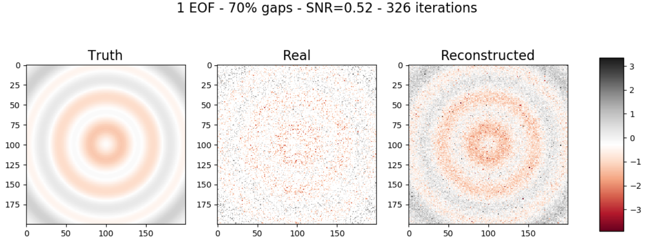
Worst case scenarii

Correlated gaps :



Random gaps :



Computation of a correlated noise

From an autocorrelation function $c(r) = r^{-\beta}$ and a white noise image b :

1. Compute power spectral density of c : $\Gamma(c) = |\mathcal{F}\{c\}|$
2. Compute FT of b : $\mathcal{F}\{b\}$
3. Do some filtering : $\mathcal{F}\{b\}\Gamma(c)$
4. Compute $\mathcal{F}^{-1}\{\mathcal{F}\{b\}\Gamma(c)\}$